

$$\frac{1}{a^8}, a^2, \quad \frac{x^2}{x^5} = \frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{1}{x^3}$$

7-4: DIVISION PROPERTIES OF EXPONENTS

$$\frac{a^6}{a^{14}} = \frac{1}{a^{14-6}} = \frac{1}{a^8}$$

Lesson Objectives:

- Divide powers with the same base
- Raise a quotient to a power

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Dividing Powers With the Same Base

PROPERTY: DIVIDING POWERS WITH THE SAME BASE

For every nonzero number a and integers m and n , $\frac{a^m}{a^n} = a^{m-n}$

a) $\frac{3^7}{3^4} = 3^{7-4} = 3^3 = 27$

b) $\frac{x^2}{x^5} = x^{2-5} = x^{-3} = \left(\frac{1}{x^3}\right)$
 $\Downarrow \frac{1}{x^{-3}} = \left(\frac{1}{x^3}\right)$

EXAMPLE 1: SIMPLIFYING A POWER RAISED TO A POWER

Simplify each expression.

1. $\frac{a^6}{a^{14}} = \frac{1}{a^{14-6}} = \left(\frac{1}{a^8}\right)$

2. $\frac{n^{-8}}{n^4} = \frac{1}{n^8 n^4}$
 $\nwarrow n^{-8-4} = n^{-12} = \left(\frac{1}{n^{12}}\right)$
 $\searrow \frac{1}{n^{4+(-8)}} = \left(\frac{1}{n^{12}}\right)$

3. $\frac{c^{15}}{c^9} = c^{15-9} = c^6$

4. $\frac{7^{-3}}{7^{-5}} = \frac{7^5}{7^3}$
 $= 7^{5-3} = 7^2 = 49$

5. $\frac{m^6}{m^{10}} = \frac{1}{m^{10-6}}$
 $= \left(\frac{1}{m^4}\right)$

6. $\frac{4^6}{4^8} = \frac{1}{4^{8-6}}$
 $= \frac{1}{4^2}$
 $= \left(\frac{1}{16}\right)$

7. $\frac{4^{-1}}{4^2} = \frac{1}{4^1 4^2} = \frac{1}{4^3}$
 $= \left(\frac{1}{64}\right)$

8. $\frac{z^7}{z^{-3}} = z^7 z^3$
 $= z^{10}$

9. $\frac{c^{-1}d^3}{c^5d^{-4}} = \frac{d^3d^4}{c^1c^5}$
 $= \left(\frac{d^7}{c^6}\right)$

10. $\frac{x^7y^9z^3}{x^4y^7z^8} = \frac{x^3y^2z^5}{z^5}$

11. $\frac{a^6b^{-5}}{a^{-2}b^7} = \frac{a^6a^2}{b^5b^7}$
 $= \left(\frac{a^8}{b^{12}}\right)$

a) $\frac{x^5}{x^7} = \left(\frac{1}{x^2}\right)$

b) $\frac{x^{-2}}{x^{-8}} = \frac{x^8}{x^2} = x^6$

12. $\frac{a^7b^5}{a^6b} = ab^4$

13. $\frac{m^8n^3}{m^{10}n^5} = \frac{1}{m^2n^2}$

14. $\frac{x^4y^{-8}z^{-2}}{x^{-1}y^6z^{-10}} = \frac{x^4x^1z^{10}}{y^8y^6y^6}$
 $= \left(\frac{x^5z^8}{y^{14}}\right)$

c) $\frac{n^{-5}}{n^2} = \frac{1}{n^5 n^2} = \left(\frac{1}{n^7}\right)$

EXAMPLE 2: DIVIDING NUMBERS IN SCIENTIFIC NOTATION

Simplify each quotient. Write each answer in scientific notation.

$$15. \frac{8 \times 10^9}{4 \times 10^5}$$

$$2 \times 10^4$$

$$\begin{array}{r} 1.9 \\ 5 \overline{)9.5} \\ \underline{-5} \\ 45 \\ \underline{-45} \\ 0 \end{array}$$

$$16. \frac{9.5 \times 10^9}{5 \times 10^4}$$

$$1.9 \times 10^5$$

$$\begin{array}{r} 0.8 \\ 8 \overline{)6.4} \\ \underline{-6} \\ 0 \end{array}$$

$$17. \frac{6.4 \times 10^9}{8 \times 10^7}$$

$$0.8 \times 10^{2-1}$$

$$8 \times 10^1$$

$$\begin{array}{r} 2. \\ 0.9 \overline{)1.8} \\ \underline{-1.8} \\ 0 \end{array}$$

$$18. \frac{1.8 \times 10^{-8}}{0.9 \times 10^{-3}}$$

$$2 \times 10^{-8-3}$$

$$2 \times 10^{-11}$$

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Raising a Quotient to a Power

PROPERTY: RAISING A QUOTIENT TO A POWER

For every nonzero numbers a and b and integer n ,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a) \left(\frac{4}{5}\right)^3 = \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{4^3}{5^3}$$

$$= \frac{64}{125}$$

EXAMPLE 3: RAISING A QUOTIENT TO A POWER

Simplify each expression.

$$19. \left(\frac{a^2}{b^3}\right)^5$$

$$\frac{a^{10}}{b^{15}}$$

$$20. \left(\frac{5a^6b^4}{c^{-3}}\right)^2$$

$$\frac{(5b^4c^3)^2}{5^2b^8c^6}$$

$$21. \left(\frac{a^8b^6}{a^{11}}\right)^5$$

$$\frac{(b^6)^5}{a^{30}}$$

$$22. \left(\frac{a^2b^{-4}}{b^2}\right)^5$$

$$\frac{(a^2)^5}{(b^4b^2)^5} = \frac{a^{10}}{b^{30}}$$

$$23. \left(\frac{a^3}{b^2}\right)^4$$

$$\frac{a^{12}}{b^8}$$

$$24. \left(\frac{2a^3b^{-2}}{c^3}\right)^5$$

$$\frac{(2a^3)^5}{b^2c^{15}} = \frac{32a^{15}}{b^2c^{15}}$$

$$25. \left(\frac{m^{-3}n^4}{n^{-2}}\right)^4$$

$$\frac{(n^4n^2)^4}{m^{12}} = \frac{n^{24}}{m^{12}}$$

$$26. \left(\frac{p^{-3}q^{-2}}{p^{-3}q^3r^5}\right)^4$$

$$\frac{(q^3)^4}{(q^3r^5)^4} = \frac{q^4}{r^{20}}$$

EXAMPLE 4: SIMPLIFYING AN EXPONENTIAL EXPRESSION

Simplify each expression.

$$27. \left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2$$

$$\frac{3^{-2}}{5^{-2}} = \frac{5^2}{3^2} \rightarrow \frac{25}{9}$$

$$28. \left(-\frac{2x}{y}\right)^{-4} = \left(-\frac{y}{2x}\right)^4$$

$$= \frac{y^4}{16x^4}$$

$$29. \left(-\frac{3}{2^3}\right)^{-2}$$

$$\left(-\frac{3}{8}\right)^{-2} = \left(-\frac{8}{3}\right)^2$$

$$= \frac{64}{9}$$

$$30. \left(\frac{x^3y^{-2}}{z^{-5}}\right)^{-4}$$

$$\left(\frac{x^3z^5}{y^2}\right)^{-4} = \left(\frac{y^2}{x^3z^5}\right)^4$$

$$= \frac{y^8}{x^{12}z^{20}}$$

$$a^0 = 1$$

$$31. \left(\frac{r^3s^{-1}}{r^2s^6}\right)^{-1}$$

$$\left(\frac{r^1}{s^7}\right)^{-1} = \frac{s^7}{r}$$

$$32. \left(\frac{2^{13}m^{14}n^{22}}{m^{-2}n^{-4}p^{33}r^{19}}\right)^0$$

$$1$$

$$33. \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4}$$

$$34. \left(\frac{s^{-4}}{t^{-1}}\right)^{-2}$$

$$\left(\frac{t}{s^4}\right)^{-2} = \left(\frac{s^4}{t}\right)^2$$

$$= \frac{s^8}{t^2}$$

$$35. \left(\frac{x^{-3}y}{xz^{-4}}\right)^{-2} = \left(\frac{yz^4}{x^3x}\right)^2$$

$$\left(\frac{yz^4}{x^4}\right)^2$$

$$\left(\frac{x^4}{yz^4}\right)^2$$

$$\frac{x^8}{y^2z^8}$$

$$\textcircled{A1}$$

$$\left(\frac{x^2y^4z^5}{x^{-1}y^2z^7}\right)^{-3} \cdot \left(\frac{x^{-2}y^2z^{-3}}{xy^{-5}z^{-5}}\right)^4$$

$$\left(\frac{x^3y^4z^5}{y^2z^7}\right)^{-3} \cdot \left(\frac{y^2y^5z^5}{x^2z^3x}\right)^4$$

$$\left(\frac{x^3y^2}{z^2}\right)^{-3} \cdot \left(\frac{y^7z^2}{x^3}\right)^4$$

$$\left(\frac{z^2}{x^3y^2}\right)^3 \cdot \left(\frac{y^7z^2}{x^3}\right)^4$$

$$\frac{z^6}{x^9y^6} \cdot \frac{y^{28}z^8}{x^{12}} = \frac{y^{22}z^{14}}{x^{21}}$$

Name _____

7-4 Practice Worksheet

Period _____

Simplify each expression.

1. $\frac{5^2 5^3}{5^3 5^2}$

2. $\frac{m^{-2}}{m^{-5}}$

3. $\frac{3^2 m^3 t^6}{3^5 m^7 t^{-5}}$

4. $\frac{4.2 \times 10^8}{7 \times 10^5}$

5. $\frac{3.5 \times 10^6}{5 \times 10^8}$

6. $\left(\frac{3a}{2b}\right)^4$

7. $\left(\frac{2p}{5}\right)^3$

8. $\left(-\frac{2}{3}\right)^{-3}$

9. $\left(\frac{3b^2}{5}\right)^2$

10. $\left(\frac{2m^5}{m^2}\right)^{-4}$

11. $\frac{(2a^7)(3a^2)}{6a^3}$

12. $\left(\frac{n^4 n}{n^{-2}}\right)^{-4}$

13. $\frac{7^9 (1^0)^2}{7^7}$

14. $\left(\frac{(3x)^2 y}{x^2 y^4}\right)^{-2}$

15. $\frac{(5a^2)(6b^3)}{(2a^3)(25b^{-2})}$

16. $\left(\frac{2ab^6}{a^3 b}\right)^{-2} \left(\frac{3a^2 b^2}{a^5 b^4}\right)^4$

Explain why each expression is *not* in simplest form.

17. $5^3 m^3$

18. $x^5 y^{-2}$

19. $(2c)^4$

20. $\frac{d^7}{d}$

21. $x^0 y$